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FREGE ON INFORMATIVE IDENTITIES BETWEEN STATEMENTS

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Abstract: The Frege-Husserl correspondence can be fruitfully explored so as to provide new insight into the paradox of analysis. Why are some identifies informative and others not? And how could we ascertain the issue if under scrutiny are mathematical identities, necessarily true if true at all? This text articulates the distinction between logical and semantic criteria in order to clarify a possible Fregean solution to the paradox of analysis, starting from regarding analysis as generating particular cases of Frege puzzles.

Keywords: paradox of analysis; Frege's puzzle; informative identities; Frege-Husserl correspondence.

1. The Problem

According to Frege, "[a]ll sentences that express a true thought have the same meaning, and all sentences that express a false thought have the same meaning (the *True* and the *False).*"[2]

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[2] Gottlob Frege, "Notes to Ludwig Darmsaedter", in *Posthumous Writings*, p. 225. In the discussion to follow, I replace "sentence" with "statement" in order to emphasize what users do with such sentences and the senses they express. As for "meaning", and its notorious ambiguity, I appeal to Fregean semantical considerations below so as to address

This is the kernel of the problem. And this is a problem not necessarily because this tenet, that all true statements are identical, is very counterintuitive, as Frege himself admits. No matter how strange this may sound (though strange, this tenet has strong arguments in its favor and many advantages) it does not mean that we could not or should not make any distinction between statements that have the same truth-value. This is a problem because this manner of seeing the reference of statements obscures the difference between informative identity statements and uninformative identity statements. In order to be able to maintain that there are informative identities – like the definition of number – one should be able to distinguish between at least three kinds of identity statements: the ones that have only their reference in common (and a completely different sense and a different linguistic expression), the ones that have their sense and their reference in common (only the linguistic expression is different), and statements that have their reference in common and have logically related-but-not-identical senses (the *informative ones*).

The focus of this essay is to find theoretical resources within the Fregean framework that enable us to distinguish between these three kinds of identity statements. On this theoretical ability rests the entire Fregean claim of being able to construct "fruitful definitions" like the definition of number.

Frege is famous for providing an elegant explanation for the difference between "trivial identities like "a=a" and informative identities like "a=b" ". The famous explanation is that the reference (*Bedeutung*) is the same but the sense (*Sinn*) is different for informative identities – which is not the case for uninformative identities. The question is: can we have the same kind of explanation for identity statements, i.e. for the case where "a" and "b" stand for propositions rather than singular terms. The difficulty is this: Singular terms were not said to have all of them the same

it. Many thanks to an anonymous reviewer for suggesting that a final account of the issue should consider Heck and May's (2020) approach. Here, however, I only sketch a possible solution to the paradox of analysis, and retrace it in the Frege-Husserl correspondence, rather than embark on an original account of meaning.

reference. Therefore, it was easy to distinguish between all kinds of different of identities. For example, it is clear how we distinguish between "Morning Star = Evening Star" and "Evening Star = planet Venus" on the one hand, and "Walter Scott = author of Waverley", on the other hand.[3] It is easy to explain why the first two identities are more closely related and have nothing to do with the third (the first two have the same object as their reference, while obviously this is not the case with the third). But it is not easy to explain in Fregean terms why identities like

a) 2(12–8) = 2 (2(6-4)) and 2(12–8) = 8

are more closely related between them than

b) 2(12–8) = 8 and another random true identity statement like 620 – 20 = 600.

In Fregean terms, all these identities have the same reference – the truth – and all have a different sense from one another, making it difficult to explain the distinction between a) and b).

2. The Significance of the Problem

To have an explanation about how and why informative identity statements differ from uninformative ones is important from at least two perspectives. First, it is important as a response to the paradox of analysis.

The paradox of analysis appears under this specific name especially in connection with G.E. Moore, namely when C. H. Langford questions Moore's notion and method of analysis.[4] Frege's own formulation of the

[3] For related discussion in Romania, see Dumitru (2004, pp. 54–55).

[4] "It is indeed possible to deny that analysis can be a significant philosophical or logical procedure. This is possible, in particular, on the ground of the so called paradox of analysis, which may be formulated as follows. Let us call what is to be analyzed the *analysandum*, and let us call that which does the analyzing, the *analysans*. The analysis then states an appropriate relation of equivalence between *analysandum* and the *analysans*. And the paradox of analysis is to the effect that, if the verbal expression representing the *analysandum* has the same meaning as the verbal expression representing the *analysans*, the

paradox of analysis can be found in his review to Edmund Husserl's *Philosophie der Arithmetik*,[5] where the dilemma is presented as a Husserlian objection:

> "If words and combinations of words mean ideas, then for any two of them there are only two possibilities: either they designate the same idea or they designate different ideas. In the former case, it is pointless to equate them by means of a definition: this is an 'obvious circle'; in the latter case is wrong. These are also the objections the author raises [i.e. Husserl], one of them regularly. A definition is also incapable of analyzing the sense, for the analyzed sense is just not the original one. In using the word to be explained, I either think clearly everything I think when I use the defining expression: we then have the 'obvious circle'; or the defining expression has a more richly articulated sense, in which case, I do not think the same thing in using it as I do in using the word to be explained: the definition is then wrong."[6]

The dilemma stems from (at least apparently) conflicting requirements: in order to be correct, the right and the left side of an identity statement should have something in common; on the other hand, in order to be non-trivial, the two sides of the identity must also incorporate significant differences. This trait of the dilemma becomes visible under a formulation preserving the general form of the problem; Michael Beaney provides one in his article "*Sinn, Bedeutung* and the paradox of analysis":

> "Consider an analysis of the form 'A is B' where A is the *analysandum* (what is analysed) and B the *analysans* (what is offered as the analysis). Then

analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect." (in C.H. Langford, "Moore's Notion of Analysis", p. 322).

[5] In 1891 Husserl's *Philosophie der Arithmetik* was published; several points from Frege's *Grundlagen* (1884) are criticized here, including the crucial definition of number by means of one-to-one correlation. Frege's reply comes in 1894, when his review of Husserl's *Philosophie der Arithmetik* was published in *Zeitschrift fur Philosophie und phil. Kritik,* vol. 103.

[6] Gottlob Frege, "Review: Husserl, Philosophy of Arithmetic" in *Collected Papers on Mathematics, Logic and Philosophy*, p. 199.

> either 'A' and 'B' have the same meaning, in which case the analysis expresses a trivial identity; or else they do not, in which case that analysis is incorrect. So no analysis can be both correct and informative."[7]

Possible ways out of the paradox will usually distinguish between two elements of the identity: one that stays the same or both sides and one element which differs from one side to another of the identity. The element which stays the same will account for the correctness of the identity relation. And the relevant difference between terms will account for the informativeness of the identity, that new piece of information obtained by acknowledging the identity. Frege's answer to Husserl's objection above follows the same patters: his distinction between *Sinn*[8] and *Bedeutung* can be seen as a possible answer to the paradox of analysis. Obviously, the identity statement would owe its correctness to the sameness of *Bedeutung* and its informativeness to the difference in *Sinn.* The aim of this essay is to see if there is a way to have a similar kind of solution for the case of identity between statements, not only between singular terms.

Secondly, the problem is important in the context of Frege's own work. It is important to see if there is an explanation for Frege's claim that certain special identity statements are more informative or "fruitful" than others.

3. A Fregean Solution: the Middle Ground

Recall that the problem is to distinguish between three kinds of identity statements with instruments provided by the Fregean framework. The aim is to delineate a certain category of identity statements that belong to the 'different sense situation' but constituted as a special class inside this category: the statements that have different-but-logically-related senses.

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[7] Michael Beaney, "*Sinn, Bedeutung* and the paradox of analysis", p. 289.

[8] I will use the German word "*Bedeutung*" each time the Fregean understanding of reference comes into discussion; the term "*Sinn*", however, will be occasionally translated as "sense".

This is what I have called "the middle ground". Finding them is the central issue here because they are the most plausible candidates for having both certitude (due to the deductive relation between their senses) and informativeness (due to the difference between their senses).

I will try to show, in what follows, that even if Frege's account of criteria for distinguishing them from the rest is not a clear and complete one, it is less hopeless than it is usually believed. At this point I will try to make clear that there were plausible reasons to maintain the opposite view, but that they are not decisive.

The 'different sense situation' can be divided into two other categories: identities that have identical reference and related senses and identities that have identical reference and completely unrelated senses. The difficulty consists in finding the appropriate criteria for distinguishing situations that involve related senses from both situations involving unrelated senses and situations involving identical senses. In other words, a middle ground must be found between sense that are "too closely related" (i.e. identical) and senses that are too far from each other (i.e. completely unrelated). The middle ground must be situated between these two extremes and, consequently, we must find criteria to distinguish it from both.

Frege offers a criterion to distinguish between two statements with related senses and two statements with unrelated senses. The criterion appears in a letter to Husserl dated 9 December 1906:

> "It seems to me that an objective criterion is necessary for recognizing a thought as the same, for without it logical analysis is impossible. Now it seems to me that the only possible means of deciding whether proposition A expresses the same thought as proposition B is the following, and here I assume that neither the two propositions contains a logically self-evident component part in its sense. If both the assumption that the content of A is false and that of B true and the assumption that the content of A is true and that of B false lead to a logical contradiction, and if this can be established without knowing whether the content of A or B is true or false, and without requiring other than purely logical laws for this purpose, then nothing can belong to the content of A as far as it is capable of being judged true or false, which does not also belong to the content of B; for there would

> be no reason at all for any such surplus in the content of B, and according to the presupposition above, such a surplus would not be logically self-evident either. In the same way, given our supposition, nothing can belong to the content of B, as far as it is capable of being judged as true or false, except what also belongs to the content of A. Thus what is capable of being judged true or false in the contents of A and B is identical, and this alone is of concern to logic, and this is what I call the thought expressed by both A and B... Is there another means of judging what part of the content of a proposition is subject to logic, or when two propositions express the same thought? I do not think so. If we have no such means, we can argue endlessly about logical questions without result."[9]

The above criterion is restated by Michael Beaney in the following abridged form:

> "(SLE) Two propositions A and B possess the same *sense* (express the same thought) iff 'both the assumption that the content of A is false and that of B true *and* the assumption that the content of A is true and that of B false lead to a logical contradiction, and ... this can be established without knowing whether the content of A or B is true or false, and without requiring other than purely logical laws for this purpose.' "[10]

The criterion makes use of the relation of equivalence between statements, but this is not the only condition it imposes. The specification that we should be able to establish that both statements have to have the same truth-value without knowing their truth-value individually is quite important. This is the condition that establishes a difference between the situation when the two statements *must* have the same truth-value and the situation when they *may* have the same truth-value by mere coincidence. This condition eliminates the possibility that the two statements have the same truth-value simply by chance, as opposed to having the same truth-value as a result of a connection between their senses. In other words, the above condition excludes the situation when

[9] Gottlob Frege, Letter to Husserl, 9 December 1906, in *Gottlob Frege. Philosophical and Mathmatical Correspondence*, pp. 70–71.
[10] Michael Beaney, *Frege: Making Sense*, p. 228.

we would first know the truth-value of one statement, then find out the truth-value of the other statement, and then we would notice that the truth-values coincide and we would declare the statements equivalent. By contrast, the situation described by the condition above is that we acknowledge the coincidence between truth-values *without* knowing which are the truth- values for each statement (i.e. we know that they must have the same truth-value, regardless of the fact that the truth-value is the false or the truth). This is the condition that reveals the dependence in sense between the two statements by 'translating' it into conditions for truth-values. The dependence in sense is tested by a simple method: in order to find out if two things are connected, we must make a change to one of them and see what happens to the other. In this case, if the two statements have related senses (i.e., if they are connected in this way, too), then the change brought to one should reverberate on the other side of the identity sign. For example,[11] in the case of

$$(462 + 864 = 1326) = (1820 + 672 = 2492)$$

there is no reason to assume that if one of them is false, so is the other. As a matter of fact, both are true. But being unrelated, one mistake on the one side would not affect in any way the other side. On the other hand, in the case of

$$(462 + 864 = 1326) = (2(231 + 432) = 1326)$$

we know that the right side and the left side must stand or fall together even *before* making the calculations in order to know if they are true or false.

This is why the criterion presented can be used to make the distinction between cases of completely unrelated senses and cases of related senses. The problem it raises, however, is the nature of this "relatedness" of senses: is this supposed to be identity or something weaker than identity? The answer emerges if we take into consideration that this criterion is presented as a criterion for *sameness of sense*.

[11] The "=" sign is, of course, used here in the same way Frege uses it, i.e. in order to express sameness of reference; in this case, sameness in truth-value.

Consequently, it makes no distinction between situations of identity of sense and senses not identical but somehow related. It cannot be used as a criterion for this distinction. Metaphorically speaking, this criterion separates the relevant identities from one extreme, but it merges them with the other extreme, namely with the 'same sense situation'. Tested, both cases react in the same way: change on the one side reverberates on the other side (in one case because of the identity, in the other because of the relatedness, in both cases, therefore, because of the common ground). It might be thought that the solution is to bring a second criterion, which would separate between the other extreme and the "middle ground"; this new criterion should be based on difference in sense instead of revealing the common ground. But Frege's criterion itself makes this approach implausible because it makes the distinction appear as illegitimate. According to the above quote, all statements that have the same sense stand in a relation of equivalence (plus the above condition) and all statements standing in a relation of equivalence (plus the above condition) have the same sense (i.e. there is a double implication between 'same sense statements' and 'equivalent statements'). Or, what I have called "middle ground", the informative identities do stand in a relation of equivalence; the resulting conclusion is that the informative identities also share the same sense.

The more serious reason why scholars assume that informative identities are 'same sense situations' rather than 'different sense situations' now becomes clear. The criterion given by Frege seems to point toward this conclusion. The reasoning is quite plausibly sound: a) the criterion for *sameness of sense* is logical *equivalence* so that equivalence implies sameness of sense and sameness of sense implies equivalence; b) between statements involved in informative identities there is a relation of equivalence. The conclusion seems unavoidable: informative identities are same sense situations.

But the Fregean account is not so straightforward in favor of this conclusion. Indeed, the Fregean account might seem contradictory. On the one hand, it is clear from the above quote that there is a double implication between 'statements having the same sense' and 'statements

in relation of equivalence'. On the other hand, Frege himself explicitly contradicts many times one direction of the double implication, namely the implication going from equivalence to sameness of sense. In other words, there is no doubt that statements that have the same sense are equivalent; the problem is if all statements that are equivalent have the same sense. Being equivalent means having the same truth-value; therefore it is clear that the latter implication contradicts Frege's assertion that many times we do have expressions with the same reference but different senses. In the case of statements, having the same reference but a different sense amounts to having the same truth-value and expressing different thoughts. This is a famous Fregean asymmetry between sense and reference: if two expressions have the same sense they cannot have different references but if two expressions have the same reference they may have different senses. Frege expresses this point of view many times:

> "We must distinguish between sense and meaning. '$2^{4'}$ and '$4^{2'}$ certainly have the same meaning, i.e. are proper names of the same number; but they have not the same sense; consequently, '$2^{4'}$ ='$4^{2'}$ and '$4\cdot4=4^{2'}$ mean the same thing, but have not the same sense (i.e., in this case: they do not contain the same thought)."[12]

Again, when connecting the asymmetry between sense and reference with the informative identities:

> "The same object can be the meaning of different expressions, and anyone of them can have a sense different from any other. Identity of meaning can go hand in hand with difference of sense. This is what makes it possible for a sentence of the form *'A = B'* to express a thought with more content than one which merely exemplifies the law of identity. A statement in which something is recognized as the same again can be of far greater cognitive value than a particular case of the law of identity. ... If in a sentence or part of a sentence one constituent is replaced by another with

[12] Gottlob Frege, "Function and Concept", in *Translations From Philosophical Writings of Gottlob Frege*, edited by Peter Geach and Max Black, Blackwell, p. 29.

> the same meaning but not with the same sense, the different sentence or part that results has the *same meaning as the original, but not the same sense.*"[13]

And in "Logic in Mathematics":

> "From this we can see that it is possible for two signs to designate the same thing and yet, because they have different senses, not to be interchangeable as far as the thought-content of sentences in which they occur is concerned."[14]

Therefore, according to these passages and according to this quite important thesis for the Fregean framework, it cannot be the case that identical references must necessarily imply identical senses. Therefore, the relation of equivalence between statements (meaning sameness of reference) cannot imply sameness of sense. Rather, the implication seems to work only in the other direction, from sense to reference (i.e. if there is sameness in sense, there is sameness in reference and, therefore, the result is the equivalence).

Accordingly, any sameness of sense implies a relation of equivalence, but equivalence between statements does not imply their sameness of sense. On the other hand, it is clear that in the passage quoted above, Frege meant a double implication when saying that "the only possible means of deciding whether proposition A expresses the same thought as proposition B" is the equivalence relation. This seems close to contradiction.[15]

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[13] Gottlob Frege, "Notes for Ludwig Darmstaedter" in *Posthumous Writings*, p. 255.

[14] Gottob Frege, "Logic in Mathematics" in *Posthumous Writings*, p. 226.

[15] This is not my original observation. It can be also found, for example, in Jean van Heijenoort's article "Frege on Sense Identity", at page 68 when commenting on the same controversial criterion for sameness of sense: "But then we are on slippery ground. In virtue of Frege's logicism, numbers can be defined in terms of logical notions, and the biconditional $(2^2 = 4) \equiv (2 + 2 = 4)$ is certainly provable by logical laws in Frege's system. Then the two sentences, '$(2^2 = 4)$" and '$(2 + 2 = 4)$', which already have the same *Bedeutung*, namely the True, would also have the same *Sinn*. In fact, we see immediately that the two sentences not containing non-logical notions would have the same *Sinn* as soon as they have the same *Bedeutung*. And, for object names (other than sentences), we would have a similar conclusion, replacing the biconditional by identity. This is an unwanted conclusion which directly contradicts what Frege says about *Sinn*. "

Choosing between the two options does not yield satisfying results either. If we accept the above criterion with all its implications and no further comment, then the consequence is that we would have to regard all statements involved in a sound proof as having the same sense; this is highly unlikely both inside and outside the Fregean framework. If, on the other hand, we reject the double implication and accept only one direction (i.e. from sameness of sense to equivalence), then we are blatantly contradicting an explicit passage from Frege.

4. Logical *versus* Semantic Criteria

However, I think that reconciliation is not impossible, even if we stay very close to the Fregean terms. A possible way out may be found in a letter to Husserl that was dated 30 October to 1 November 1906. The way I am interpreting this text may bring coherence into this whole divided picture. The main idea is that the distinction between 'same sense'/ 'different sense' situations is simply not accessible in certain contexts. Metaphorically speaking, it might be said that a division in the area of equivalent statements between same sense and different sense statements is not "visible" from a certain point of view. This point of view is, for Frege, the logical point of view (i.e. the objective and scientific point of view). According to this interpretation, the problem with the same sense/different sense distinction is that it is a *semantic* distinction, visible and present in the natural language, but untranslatable in terms of truth-values and combinations of truth-values (i.e. inexpressible by purely logical means).

In the above mentioned letter, Frege speaks about "equipollent propositions", what we would today call logically equivalent propositions, and about the impossibility of objectively distinguishing between "merely equipollent and congruent propositions" (i.e. between propositions that have the same truth-value and the ones that share more than their truth-value, namely also their sense):

> "One should make only those distinctions with which the laws of logic are concerned. In gravitational mechanics no one would want to distinguish

> bodies according to their optical properties. ... In logic one must decide to regard equipollent propositions as differing only according to form. After the assertoric force with which they may have been uttered is subtracted, equipollent propositions have something in common in their content, and this is what I call the thought they express. This alone is of concern of logic. The rest I call the colouring and the illumination of the thought. Once we decide to take this step, we do away at a single stroke with a confused mass of useless distinctions and with the occasion for countless disputes which cannot for the most part be decided objectively. And we are given free reign to pursue proper logical analyses. Judged psychologically, the analyzing proposition is, of course always different from the analyzed one, and all logical analysis can be brought to a halt by the objection that the two propositions are merely equipollent, if this objection is indeed accepted. *For it will not be possible to draw a clearly recognizable limit between merely equipollent and congruent propositions.*[16] Even propositions which appear congruent when presented in print can be pronounced with a different intonation and are not, therefore equivalent in every respect. Only now that logical analysis proper has become possible can the logical elements be recognized, and we can see the clearing in the forest. ... It cannot be the task of logic to investigate language and determine what is contained in a linguistic expression." [17]

It appears from this passage that "equipollent propositions" have in common the thought they express. This is, again, the expression of the above incriminated thesis that equivalence implies sameness of sense (or of thought, respectively). But this affirmation is made after one important specification: it is *in logic* that we must so construe senses. It is also relevant that this is presented as a prescription and a practical decision: we must *decide* to see things this way, it is not the situation that simply presents itself so. One more important specification is made: any difference other than in truth-value must be completely entrusted to the form of the expression (i.e. to the formalized language of logic): "In logic one must decide to regard equipollent propositions as differing only

[16] My italics.

[17] Gottlob Frege, "Letter to Husserl, 30 October to 1 November 1906" in *Philosophical and Mathematical Correspondence*, p. 67.

according to form." We might derive from this the opinion that any difference in sense, if it cannot be translated in terms of truth-values, should appear in the logical form of the expression; anything else would be the mere "illumination" of the thought.

My interpretation of the passage is that equivalent statements are considered to be, all of them, "same sense" situations because the difference in sense cannot be reflected in logic: this difference has no influence upon the distribution of truth-values. Consequently, Frege considers that there are no *logical* means to make this difference, though in *semantic* terms, the difference can be made. Logic, in order to be "topic neutral" and because "it cannot be the task of logic to investigate language", cannot mirror the difference between senses if this difference shows neither in the combinations of truth-values nor in the formal expression. In the case of the distinction between same sense/different sense, this "mirroring" does not take place because senses are, basically, *ways of obtaining* a certain result, not the result itself. The combination in truth-values can correspond only to the result, not to the way the result is obtained; from the point of view of truth-values, only the result matters and therefore, from this point of view the result is the *same* irrespective of the way in which it is attained. My interpretation of Frege's affirmation – that "the only possible means of deciding whether proposition A expresses the same thought as proposition B" is the relation of equivalence – amounts to saying that the difference between same sense/different sense situations cannot be expressed because this difference cannot be translated in a *calculus* manipulating truth-values. Senses, as ways of obtaining certain results, cannot enter the calculus; only their results can.

This version appears to be confirmed by Frege's affirmation in the sequel of the same letter, when he answers the question if two statements are "merely equipollent" or "equipollent and congruent". The statements are "If A then B" and "It is not the case that A without B", i.e. what today we would represent as 'A→B' and '~ (A& ~B)'. Frege's verdict is that they are equipollent (i.e. equivalent) but that nothing else can be said from the point of view of logic (i.e. the scientific, certain point of view):

> "In each case we therefore have an equipollence. ... Now are these propositions also congruent? This could well be debated for a hundred years or more. At least I do not see what criterion would allow us to decide this question objectively. But I do find that if there is no objective criterion for answering a question, then the question has no place at all in science."[18]

Against this very emphatic dismissal of the problem for the domain of logic it can be said that the same problem, of sameness and difference, can reappear on strictly logical grounds. For the significant difference in sense cannot be completely entrusted to the difference in notation or to the difference in formalization; some logical formulas can be regarded as mere redundant re-writings of other formulas, while other as genuine inferential transformations. So that the problem of distinguishing between mere re-writings/ inferential transformation can appear also in logic (i.e. it is not a problem of "investigation of language") and, possibly, may have a strictly logical answer by means of calculus in terms of truth-values.

This first criterion is the most problematical one. The second criterion, from "A Brief Survey of my Logical Doctrines", is less problematic, but it is not a *logical* one; it is rather, an epistemological one: "anyone who recognizes the content of A as true must straight away also recognize that of B as true, and conversely, anyone who recognizes the content of B must immediately also recognize that of A".

The usefulness of his criterion for the present purpose resides in the fact that it can be used to make the distinction discussed above: it can be used as a test for identities so that the informative identities can be distinguished from the other extreme, from the situation of identities with the same sense.

The challenge was to find a way to distinguish the "middle ground" from both extremes: one of the same sense situations and one of completely unrelated senses. The first criterion, amalgamating the "middle ground" with the 'same sense' extreme, was capable to establish

[18] Gottlob Frege, "Letter to Husserl, 30 October to 1 November 1906" in *Philosophical and Mathematical Correspondence*, pp. 68–69.

a difference from the situation of completely unrelated senses, but not from the other extreme. This second criterion, though it does not satisfy Frege's strict requirement in that it does not use strictly logical means, can separate between 'same sense identities' and 'different but related in sense' identities. This separation is made by the fact that, if this criterion is taken as a test, then the 'different but related in sense' identities *fail* it. When there is a difference in sense between A and B, two statements standing in the relation of identity, then we do not *immediately* recognize that if A is true, also B must be true (and the converse). It is only a *mediated* recognition, namely mediated by inferences or by further factual information. The *immediat*e recognition can be made only in the case of strict sameness of sense.

In conclusion, we may say that in the case of the "middle ground" identities, unlike the case of the 'unrelated sense' identities, a change in the one side of the identity will reverberate on the other side; and unlike the case of 'same sense' identities, we might *not recognize* that *immediately* (because of the necessary mediation).

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